

The Power Set of a Set

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Definition

Definition: If A is a set, the **power set** of A , written $\mathcal{P}(A)$, is the set whose elements are all subsets of A . In set builder notation,

$$\mathcal{P}(A) = \{X : X \subseteq A\}$$

Example

$$A = \{0, 1, 3\}$$

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$$\mathcal{P}(A) = \{\emptyset, \{0\}, \{1\}, \{3\}, \{0, 1\}, \{0, 3\}, \{1, 3\}, \{0, 1, 3\}\}$$

$$\wp(\wp(A))$$

Example

$$\mathcal{P}(\emptyset) = \{\emptyset\}$$

Notice that $|\emptyset| = 0$ and $|\mathcal{P}(\emptyset)| = 2^0 = 1$.

Example

$$\mathcal{P}(\{a\}) = \{\emptyset, \{a\}\}$$

Example – some common mistakes

$\mathcal{P}(1)$ makes no sense because 1 is not a set.

Example – some common mistakes 2

$\mathcal{P}(\{1, \{1, 2\}\}) = \{\emptyset, \{1\}, \{\{1, 2\}\}, \{1, \{1, 2\}\}\}$. Notice that $\{1, 2\}$ is not an element of $\mathcal{P}(\{1, \{1, 2\}\})$ but $\{\{1, 2\}\}$ is.

$$\{1, 2\} \in \{1, \{1, 2\}\}$$

$$\{\{1, 2\}\} \subseteq \{1, \{1, 2\}\}$$

Infinite case

The power set $\mathcal{P}(\mathbb{N})$ is very large and can be identified with infinite sequences of 1's and 0's.

$$\begin{array}{ccccccc} A & 1 & 0 & 0 & 0 & 1 & \dots \\ \cap \\ \mathbb{N} = \{ & 1 & 2 & 3 & 4 & \dots & \} \end{array}$$

The set $\mathcal{P}(\mathbb{R}^2)$

$\mathcal{P}(\mathbb{R}^2)$ is huge and includes every graph of every function plus lots of other things, more than we can really comprehend.

Problem 1.4.15

What is $\mathcal{P}(A \times B)$ if $|A| = m$ and $|B| = n$?

$$A = \{1, 2\} \quad B = \{a, b, c\}$$

$$(1, a) \in A \times B$$

$$(2, b) \in A \times B$$

$$\{(1, a), (2, b)\} \in \mathcal{P}(A \times B)$$

$$2^{mn} = |\mathcal{P}(A \times B)| = |\mathcal{P}(A) \times \mathcal{P}(B)| = 2^{m+n}$$

$$\mathcal{P}(A) \times \mathcal{P}(B)$$

$$\{2\} \in \mathcal{P}(A)$$

$$\{a\} \in \mathcal{P}(B)$$

$$\{2\} \times \{a\} = \{(2, a)\}$$

$$\{1, 2\} \subseteq A$$

$$\{b, c\} \subseteq B$$

$$A \times B$$

$$\{(1, b), (1, c), (2, b), (2, c)\}$$